

Writing Quadratic Equations

What Goes Up Must Come Down

ACTIVITY 10

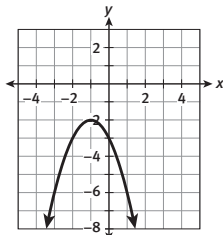
continued

ACTIVITY 10 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 10-1

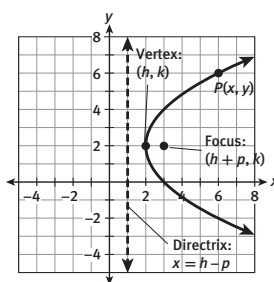
Use the parabola shown in the graph for Items 1 and 2.



- What is the equation of the parabola?
A. $y = -(x-1)^2 - 2$ **B.** $y = -(x+1)^2 - 2$
C. $y = (x-1)^2 - 2$ **D.** $y = (x+1)^2 + 2$
- The focus of the parabola is $(-1, -\frac{9}{4})$, and the directrix is the line $y = -\frac{7}{4}$. Show that the point $(-2, -3)$ on the parabola is the same distance from the focus as from the directrix.
- Graph the parabola given by the equation $x = \frac{1}{2}(y-3)^2 + 3$.
- Identify the following features of the parabola given by the equation $y = \frac{1}{8}(x-4)^2 + 3$.
a. vertex **b.** focus
c. directrix **d.** axis of symmetry
e. direction of opening
- Describe the relationships among the vertex, focus, directrix, and axis of symmetry of a parabola.
- The focus of a parabola is $(3, -2)$, and its directrix is the line $x = -5$. What are the vertex and the axis of symmetry of the parabola?

For Items 7–11, use the given information to write the equation of each parabola.

- vertex: $(0, 0)$; focus: $(0, 5)$
- vertex: $(0, 0)$; directrix: $x = -3$
- vertex: $(2, 2)$; axis of symmetry: $y = 2$; focus: $(1, 2)$
- opens downward; vertex: $(-1, -2)$; directrix: $y = -1$
- focus: $(-1, 3)$; directrix: $x = -5$
- Use the diagram below to help you derive the general equation of a parabola with its vertex at (h, k) , a horizontal axis of symmetry, a focus of $(h+p, k)$, and a directrix of $x = h-p$. Solve the equation for x .



Lesson 10-2

Write the equation of the quadratic function whose graph passes through each set of points.

- $(-3, 0), (-2, -3), (2, 5)$
- $(-2, -6), (1, 0), (2, 10)$
- $(-5, -3), (-4, 0), (0, -8)$
- $(-3, 10), (-2, 0), (0, -2)$
- $(1, 0), (4, 6), (7, -6)$
- $(-2, -9), (-1, 0), (1, -12)$

ACTIVITY 10 Continued

ACTIVITY PRACTICE

1. B

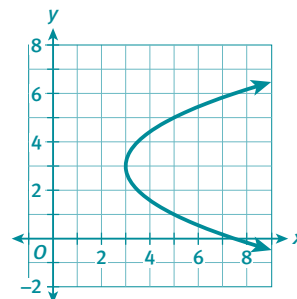
2. distance to focus:

$$\sqrt{(-1 - (-2))^2 + \left(-\frac{9}{4} - (-3)\right)^2} = \frac{5}{4}$$

distance to directrix:

$$\sqrt{(-2 - (-2))^2 + \left(-\frac{7}{4} - (-3)\right)^2} = \frac{5}{4}$$

3.



- $(4, 3)$
 - $(4, 5)$
 - $y = 1$
 - $x = 4$
 - upward
- Sample answer: The axis of symmetry is perpendicular to the directrix. The focus and the vertex lie on the axis of symmetry. The vertex is the midpoint of the segment that lies on the axis of symmetry and has its endpoints at the focus and on the directrix.
 - Vertex: $(-1, -2)$; axis of symmetry: $y = -2$
 - $y = \frac{1}{20}x^2$
 - $x = \frac{1}{12}y^2$
 - $x = -\frac{1}{4}(y-2)^2 + 2$
 - $y = -\frac{1}{4}(x+1)^2 - 2$
 - $x = \frac{1}{8}(y-3)^2 - 3$
 - $y = x^2 + 2x - 3$
 - $y = 2x^2 + 4x - 6$
 - $y = -x^2 - 6x - 8$
 - $y = 3x^2 + 5x - 2$
 - $y = -x^2 + 7x - 6$
 - $y = -5x^2 - 6x - 1$

12. Sample derivation:

distance from P to focus = distance from P to directrix

$$\sqrt{(x - (h+p))^2 + (y - k)^2} = \sqrt{(x - (h-p))^2 + (y - y)^2}$$

$$(x - (h+p))^2 + (y - k)^2 = (x - (h-p))^2 + (y - y)^2$$

$$x^2 - 2(h+p)x + (h+p)^2 + (y - k)^2 = x^2 - 2(h-p)x + (h-p)^2$$

$$x^2 - 2hx - 2px + h^2 + 2hp + p^2 + (y - k)^2 = x^2 - 2hx + 2px + h^2 - 2hp + p^2$$

$$-2px + 2hp + (y - k)^2 = 2px - 2hp$$

$$(y - k)^2 + 4hp = 4px$$

$$\frac{1}{4p}(y - k)^2 + h = x$$

ACTIVITY 10 Continued

19. Answers may vary, but equations should be a nonzero multiple of $y = x^2 + 2x - 48$. Sample answer: The parabolas given by the equations $y = x^2 + 2x - 48$ and $y = -x^2 - 2x + 48$ both pass through the points $(-8, 0)$ and $(6, 0)$.
20. a. $(-1, 5)$. Sample explanation: For a quadratic function, the axis of symmetry is a vertical line that passes through the vertex, so the axis of symmetry is $x = 3$. The point $(7, 5)$ is 4 units to the right of the axis of symmetry, so there will be another point on the graph of the function 4 units to the left of the axis of symmetry with the same y -coordinate. This point has coordinates $(-1, 5)$.
- b. $f(x) = \frac{1}{4}x^2 - \frac{3}{2}x + \frac{13}{4}$
21. Sample justification: A linear model is a better fit. The values of y increase as x increases without ever decreasing, which indicates the shape of a linear, not a quadratic, model. Linear model: $y = 4.9x + 18.2$
22. Sample justification: A quadratic model is a better fit. A graph of both models shows that the data points are closer to the quadratic model. Also, the values of y first decrease and then begin to increase as x increases, which indicates the shape of a quadratic, not a linear, model. Quadratic model: $y = 0.3x^2 - 5.0x + 23.8$
23. car: $y = 0.047x^2 + 2.207x + 0.214$; truck: $y = 0.064x^2 + 2.210x - 0.500$
24. Predictions should be close to 42 feet.
25. No. Sample explanation: Based on the quadratic model, the stopping distance for the truck at 60 mi/h is about 363 feet. This distance is greater than the distance between the truck and the intersection, so the driver will not be able to stop in time.
26. a. $y = -2.2x^2 + 454.9x - 12,637.0$
- b. Yes. Sample explanation: A graph of the quadratic model and the data from the table shows that the graph of the model is close to the data points. Also, the monthly revenue increases and then decreases as the selling price increases, which indicates a quadratic model could be a good fit for the data.
- c. Answers should be close to \$103.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the Teacher Resources at SpringBoard Digital for additional practice problems.

ACTIVITY 10

continued

19. Demonstrate that the points $(-8, 0)$ and $(6, 0)$ do not determine a unique parabola by writing the equations of two different parabolas that pass through these two points.
20. a. The graph of a quadratic function passes through the point $(7, 5)$. The vertex of the graph is $(3, 1)$. Use symmetry to identify another point on the function's graph. Explain your answer.
- b. Write the equation of the quadratic function.

Lesson 10-3

Tell whether a linear model or a quadratic model is a better fit for each data set. Justify your answer and give the equation of the better model.

21.

x	0	2	4	6	8	10	12	14
y	17	29	40	45	59	63	76	88

22.

x	2	4	6	8	10	12	14	16
y	15	9	5	2	6	7	16	22

The stopping distance of a vehicle is the distance the vehicle travels between the time the driver recognizes the need to stop and the time the vehicle comes to a stop. The table below shows how the speed of two vehicles affects their stopping distances.

Speed (mi/h)	Stopping distance (ft)	
	Car	Truck
10	27	28
15	44	47
20	63	69
25	85	95
30	109	123
35	135	155
40	164	190

23. Use a graphing calculator to perform a quadratic regression on the data for each vehicle. Write the equations of the quadratic models. Round coefficients and constants to the nearest thousandth.

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24. Use your models to predict how much farther it would take the truck to stop from a speed of 50 mi/h than it would the car.
25. Suppose the truck is 300 ft from an intersection when the light at the intersection turns yellow. If the truck's speed is 60 mi/h when the driver sees the light change, will the driver be able to stop without entering the intersection? Explain how you know.

MATHEMATICAL PRACTICES

Use Appropriate Tools Strategically

26. A shoe company tests different prices of a new type of athletic shoe at different stores. The table shows the relationship between the selling price and the monthly revenue per store the company made from selling the shoes.

Selling Price (\$)	Monthly Revenue per Store (\$)
80	9680
90	10,520
100	11,010
110	10,660
120	10,400
130	9380

- a. Use a graphing calculator to determine the equation of a quadratic model that can be used to predict y , the monthly revenue per store in dollars when the selling price is x dollars. Round values to the nearest tenth.
- b. Is a quadratic model a good model for the data set? Explain.
- c. Use your model to determine the price at which the company should sell the shoes to generate the greatest revenue.